Using EXCEL for numerical analysis of fluid flow in series and parallel piping systems and discussing the limitations of the numerical techniques

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I: Abstract

Fluid dynamics is governed by Bernoulli's equation which is based on the principle of conservation of energy. Bernoulli's equation is an energy balance equation that converts fluid potential energy, kinetic energy, and stored energy due to pressure to an equivalent head for a single branch fluid flow system. The Head is in units of length. Friction losses can also be expressed in terms of head and be included in Bernoulli's equation. However, if the flow rate is unknown, there are more unknowns than equations, and Bernoulli's equation cannot be used directly. In such a scenario the fluid system must be analyzed by iterative techniques. In the case of a series system, there is only one flow rate and the flow rate can be incrementally changed until the correct solution is obtained. However, in a parallel system, the flow rates are different in various branches. For the parallel piping systems, a halving technique where the interval is cut into half on a repeated basis can be used to investigate various flow rates until the correct flow rate with sufficient accuracy is determined. An EXCEL spreadsheet can be used to automate this process. The analysis techniques for both the series piping systems and parallel piping systems are the theme of this article.

II: Introduction

Flow analysis for fluids in all piping systems is performed based on energy principles. The total energy of the fluid is determined by a term called head which has units of length. But, in reality, the head is energy per unit weight, and the term head in units of length is used to enable the use of a single unit for sources of fluid energy which are fluid velocity, fluid height, and fluid pressure. For the reason of consistency of units, energy losses due to friction and added energy due to pumping action are also converted into units of length.

Friction losses in fluids are a function of fluid velocity, fluid viscosity, and pipe roughness. In a series piping system, if either the flow rates or the pipe sizes are not known, there will be more unknowns than the number of equations. In such a scenario, iterative techniques can be used, or alternatively plots with a significant number of assumed velocities can be made to find a solution. EXCEL can be used to automatically generate the plots. In parallel piping systems, the flows are divided among the various branches in such a manner that the head loss in all branches is the same. Finding a solution for the parallel system also requires iterations. EXCEL combined with sorting techniques borrowed from elementary sorting algorithms can be used to expedite the process of getting solutions for parallel systems.

In this article, solution techniques for both series and parallel systems are discussed and examples are presented.

III: Nomenclature

P: Pressure
γ: Specific Weight
V: Velocity
g: Specific gravity constant
Z: Elevation
h: Head
K: head loss coefficient
L: Pipe length
D: Pipe diameter
f: Friction factor
N_R: Reynold's number
ν: Kinematic viscosity
Q: Fluid flow rate

IV: Discussion of series piping systems

Figure 1 is an example of a series piping system that will be used to illustrate the use of EXCEL for analyzing series piping systems. [2]

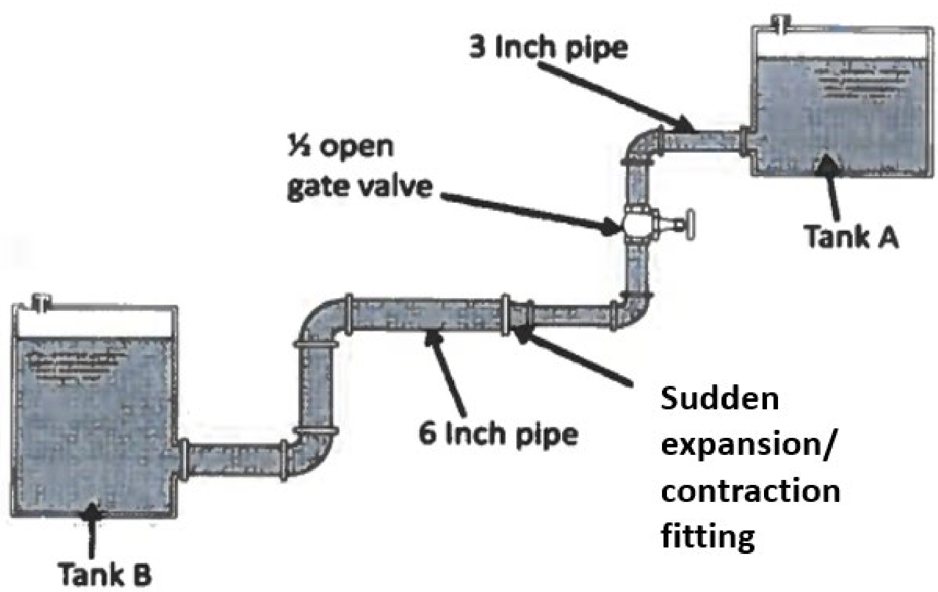

Figure 1: Example of s series piping system

The three sources of fluid energy are pressure, height, and velocity. Height and velocity can readily be put in terms of energy. However, pressure cannot be put in terms of energy directly. In order to have consistent units of energy from the three sources, all terms are converted to head which is in units of length. However, in reality, the head is energy per unit mass of fluid. Losses are also put in units of the head. In a series piping system, the use of the head in units of length is sufficient because the flow rate does not change. However, as it will be presented in section V of this document, for a parallel system the true units of energy per unit mass must be used. The following formulas (1 through 6) define the heads from various sources.

Pressure head = P/γ (equation 1)
Velocity head = $V^2 / 2g$ (equation 2)
Elevation head = h (equation 3)
Loss due to friction = $h_L = K (V^2 / 2g)$ (equation 4; K is determined based on fluid viscosity, fluid velocity, and pipe roughness from equation 5).
$K = fL / D$ (equation 5; f is determined from equation (6) for turbulent flow). [2]
$f= 0.25 / \{[\log (1/3.7 (D/\epsilon)) + (5.74/ N_R^{0.9})]^2\}$ (equation 6; N_R is determined from equation 7).
$N_R = VD/ \nu$ (equation 7)
Miscellaneous losses = $h_L = K (V^2 / 2g)$ (equation 8; K is determined based on experimental values for elbows, valves, etc.)

The heads can then be put into an energy balance equation as shown in equation (9).

$P_1/\gamma + (V_1^2/2g) + Z_1 - h_L = P_2/\gamma + (V_2^2/2g) + Z_2$ (equation 9)

In equation (9), points 1 and 2 are 2 points of interest from an energy balance point of view.

If the flow rate is known, equation (9) can directly be used as shown in example 1. However, if the flow rate is not known, there are more unknowns than knowns because friction is dependent on fluid velocity. When equation (9) cannot be used directly, an iterative approach to the use of equation (9) must be used as shown in example 2.

Example 1:

For the system in figure 1, calculate the vertical distance between the surfaces of the two reservoirs when fluid flows from tank A to tank B at a rate of 0.03 ft^3/sec. The inside diameter of the 3-inch pipe is assumed to be 3 inches, and the inside diameter of the 6-inch pipe is assumed to be 6 inches. The total length of the 3-inch pipe is 330 ft and the total length of the 6-inch pipe is 990 ft. Fluid kinematic viscosity is assumed to be 14×10^{-6} ft^2/sec.

The energy balance relationship shown in equation (9) can be used. Since the velocities at the fluid top in both tanks are zero(0), the velocity terms drop out. The pressure at the top of the fluid level in both tanks is atmospheric, and consequently, the pressure terms of equation (9) also drop out. Therefore, the required height is the height that can overcome the total frictional losses expressed in units of length (head). The minimum required height difference is calculated to be .31 ft.

Example 2:

Create a table of height difference versus flow rate for the system of example 1 using EXCEL. Then use the EXCEL curve fitting feature to obtain an approximate expression relating flow rate to the fluid height difference.

Table 1 contains the data for example 2.

Table 1: Selected flow rate versus height difference for the piping System of figure 2

Height difference between tanks A & B (Inch)	Flow rate (ft^3/sec)
0	0
1.8	0.02
6.3	0.04
13.2	0.06
22.4	0.08
34.1	0.1
48.0	0.12
64.3	0.14
82.9	0.16
103.8	0.18
127.0	0.20
152.5	0.22
180.3	0.24

Figure 2 shows the data of table 1, and a second-degree polynomial fit obtained from EXCEL. Figure 2 shows that the second-degree polynomial fit produces a satisfactory approximate formula for a particular piping system such as the one shown in figure 1. In reference [1] it is shown that satisfactory formulas for more complex systems can be obtained using the curve fitting feature of EXCEL. In figure 2, "Series 1" is the data from table 1, and "series 2" is the curve fitting plot. The formula for the curve fitting plot is shown in formula (10). In formula (10), HD is the height difference between the fluid tops in tanks A and B.

$Q= -5\ (10)^{-6}\ (HD)^2 + 0.0021\ (HD) + 0.249$ (equation 10)

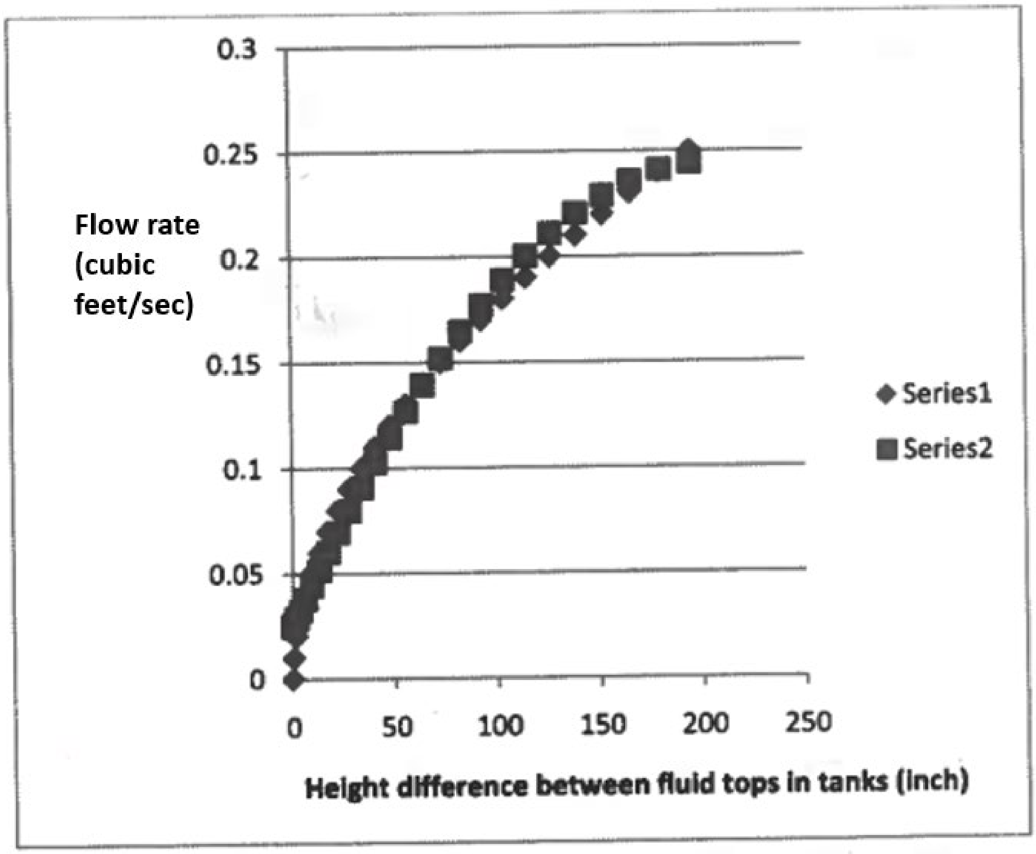

Figure 2: Plot of data of table 1 and data obtained from curve
Fitting feature of EXCEL.

V: Discussion of parallel piping systems

Figure 3 is an example of a parallel piping system with 3 branches. [2]

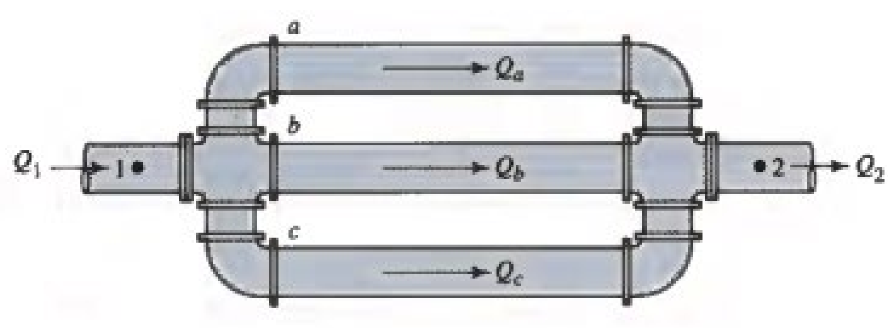

Figure 3: Example of a series piping system

Equation (9) which is an energy balance equation for fluid systems applies to parallel piping systems as well as series piping systems. This form of energy equation states that the difference in pressures between points 1 and 2 of the system in figure 3 depends on elevation difference, the difference in the velocity heads, and the energy loss per unit mass of fluid due to friction. When any element of fluid in figure 3 reaches point 2, each experiences the same elevation change, the same velocity change, and the same energy loss per unit weight regardless of the path. Consequently, all fluid flows in all branches reaching point 2 have the same head. This can be mathematically stated as shown in equation (11). [2]

$h_{L\ 1\text{-}2} = h_a = h_b = h_c$ (equation 11)

The total flow at points 1, 2 and braches of the system of figure 3 result in equation (12).

$Q_1 = Q_2 = Q_3 = Q_a + Q_b + Q_c$ (equation 12)

When three or more parallel branches occur in a pipe flow system, the system is indeterminate because there are more unknowns than independent equations. In such a scenario, one technique is an iterative process. A quicker alternative is that flows can be assumed and plotted using EXCEL and when the data converge would show the flow distribution among the parallel branches.

VI: Student end of semester reactions

Engineering Technology in the author’s institution is intended not to be mathematically oriented but to be practical and design-oriented. The techniques presented in this article have been used in an Engineering Technology Fluid Mechanics course because it is mathematically simple and is simple to program in EXCEL. The mathematical and programming simplicity don’t divert students’ attention from the course focus which is the design of fluid systems. The students were given feedback regarding their computer models and then assisted until their models were producing the correct results. At end of the semester, student comments indicated the technique had helped them to better understand the concepts**.**

VIII: Summary and conclusion:

In this article, the use of the EXCEL spreadsheet and its graphical capabilities were demonstrated as an aid in determining the flow rate in series and parallel piping systems. In the case of series piping systems, the dependency of friction losses and flow velocity leads to more unknowns than a number of independent equations. Curve fitting features of EXCEL were presented as an alternative numerical technique for solving such problems. In parallel systems, when there are three or more parallel branches, there are more unknowns than the number of independent equations. Graphical capabilities of EXCEL were suggested as an alternative to numerical techniques. These techniques were used as lab assignments in fluid courses in the author’s institution. Student feedback regarding the usefulness of the techniques as related to improving students’ understanding of the subject matters was positive.

IX: References

[1]. Using EXCEL for enhancing analysis of series piping systems by Hagigat, Computers in education journal published by computers in education division of ASEE, Vol 2 No3 July – September 2011.

[2]. Applied Fluid Mechanics, Sixth Edition, Author: Robert L. Mott.

[3]. Engineering Fluid Mechanics, by Crowe, Elger, Williams and Roberson.