Comparing Stress results obtained by using computerized Finite Element Analysis Techniques and Classical Stress Analysis Techniques

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Abstract:
Using the theory of elasticity and advanced mathematics, numerous deflection and stress formulas have been developed for geometries and loading conditions that fall outside of the scope of classic strength of material. These solutions are referred to as classical stress formulas. Roark's formulas for stress and strain is the standard document that has gathered the classical stress formulas in one place.

Finite element (FE) modeling is also a technique that is widely used for deflection and stress calculation of geometric and loading conditions that fall outside the scope of classical strength of material. The FE technique can model any geometry and loading condition. However, the accuracy of the FE model is dependent on the modeling technique.

The theme of the current article is to verify the FE modeling technique by developing FE models for geometric and loading conditions that have known classical stress formulas that are also close to the geometric and loading conditions of the FE model in order to verify the FE modeling technique and then making minor adjustments to the verified FE model in order to assure its accuracy.

The techniques demonstrated in the current article were used in a Machine Design course and have possible future use in Senior Capstone design projects at the author's institution.

Keyword

- Finite Element analysis
- Classical Stress analysis
- NASTRAN/PATRAN
- Structural Analysis

Introduction:
Roark's formulas for stress and strain reference textbook has numerous deflection and stress formulas pertaining to numerous geometric and loading conditions. It is an accepted fact that Roark's formulas are correct and accurate. A FE modeling technique is confirmed to be accurate

by comparing its results to known Roark's formulas. Once a modeling technique is confirmed to be correct, it can then be subjected to minor changes involving geometry, boundary conditions, and loading and its results will be correct.

One of the formulas in Roark's is for a thin circular plate that is either simply supported or fixed at its edge that is subjected to an axisymmetric uniform pressure either on its entire surface or starting from a certain radius to its edge. The current article shows that a Finite Element (FE) model developed using NASTRAN/PATRAN software can generate accurate results that are close to Roark's, or in the case of improper modeling practices will generate incorrect results.

Overview of supporting literature:

The 6th edition of Roark's Formulas for Stress and Strain[1] is written for the purpose of making a compact summary of the formulas pertaining to geometric and loading conditions that cannot be easily solved by techniques presented in traditional strength of materials textbooks.[2] The formulas in Roark's have been developed by innovative application of elasticity theory and by using advanced mathematical techniques.

Finite Element analysis techniques can also be used for structural analysis of non-standard geometric and loading scenarios. The Finite Element technique discretizes the structure into small segments, and if sufficient and correct small segments are used, accurate deflection and stress values will result.[3]

There are a number of commercially available Finite Element software. NASTRAN/PATRAN is one of the more widely used Finite Element software. NASTRAN/PATRAN provides a number of techniques for analyzing non-standard geometric and loading techniques.[4] By using NASTRAN/PATRAN software, a finite element-based solution can be obtained for any loading condition and geometry. However, the accuracy of the Finite Element technique must be verified.

Description of educational approach of combining classical stress analysis techniques and Finite Element analysis method:

In a Machine Design course, various structural failure theories are used. All failure theories are based on some sort of stress analysis of the part. The author used Roark's formulas and the Finite Element technique in a machine design course and demonstrated the similarities and differences between the results obtained. The theme of this article is verifying the accuracy of stress analysis results by combining Roark's formulas and the Finite Element technique.

Example of use of Roark's formulas for stress analysis of a thin uniformly loaded circular plate:

Table 24 of Roark's contains stress formulas for a circular plate subjected to a uniformly distributed axisymmetric load from an inner radius to its edge. Figure 1 illustrates the geometry and the loading condition. The inner radius can be zero, which means the uniform pressure is applied to the entire circular plate, or it can start at a certain radius r_0.

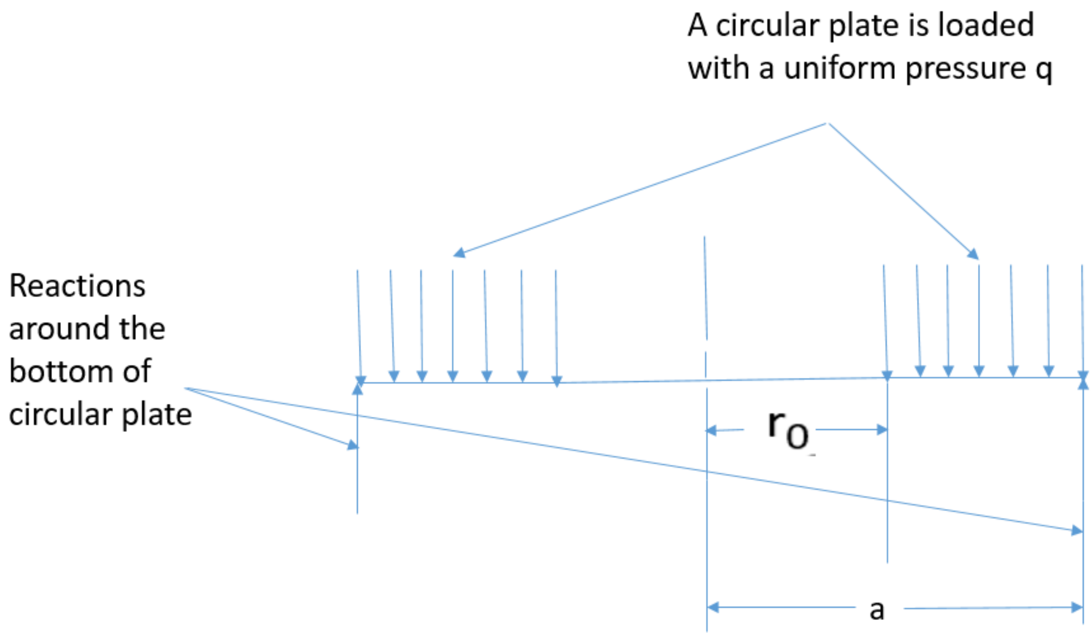

Figure 1: Side view a circular plate supported at its edge subjected to a uniformly distributed pressure

The following notation and formulas apply to the Roark's based solutions that follow.
v = Poisson ratio
t= plate thickness
E= Modulus of Elasticity
$D = (E\ t^3) / \{12\ (1 - v^2)\}$
"ln" sands for natural log in all the above formulas.

$L_{11} = \{1 + (4(r_0/a)^2)\text{-}(5(r_0/a)^4) - [(4(r_0/a)^2)[2 + (r_0/a)^2]\ln(a/r_0)]\}/64$

$L_{14} = \{1\text{-}\ (r_0/a)^4 - (4\ (r_0/a)^2(\ln(a/r_0)))\}/16$

$L_{17}= \{1\text{-}(((1\text{-}v)/4)(1\text{-}(r_0/a)^4)) - ((r_0/a)^2(1 + ((1 + v)\ \ln(a/r_0))))\}/4$

The following formulas apply to a simply supported circular plate where the uniform pressure does not start at the center of the circular plate. Instead, the uniform pressure is applied from a radius (r_0 of figure 1) to the plate edge in an axisymmetric manner.

Deflection at plate center = $y_c = (qa^4/2D)\{[L_{17} / (1 + v)] - 2L_{11}\}$
Bending moment at center of plate = $M_c = qa^2L_{17}$ (Equation 1)

The formulas that follow apply to a simply supported circular plate where the uniform pressure is applied to the entire plate (from center to edge).

Deflection at center of plate = y_c= {q a^4 (5 + ν)} / {64 D (1 + ν)}

Bending moment at center of plate= M_C = {q a^2 (3 + ν)} / 16 (Equation 2)

The following formulas apply to a plate where the plate is fixed at its edge, and the uniform pressure is either applied to the entire pressure or is applied from a radius (r_0 of figure 1) to the plate edge in an axisymmetric manner. By setting the value of r_0 to zero, the formulas represent the uniform pressure being applied to the entire surface.[4]

Deflection at the center of the plate= y_c = (qa/2D)(L_{14} – 2 L_{11})

Bending moment at center of plate = M_C = q a^2 (1 + ν) L_{14} (Equation 3)

Bending moment at plate edge= M_{ra} = (q/8a^2)(a^2 – r_0^2)^2 (Equation 4)

The following formulas apply to a circular plate fixed at its edge where the uniform pressure is applied to the entire plate (from center to edge).

Deflection at center of plate = y_C = {qa^4} / {64D}

Bending moment at center of plate = M_C = {q a^2 (1 + ν)} / 16 (Equation 5)

Bending moment at plate edge= M_{ra} = qa^2/8 (Equation 6)

The stress due to bending at any point on the plate is governed by the following equation where M is the bending moment at the location of interest and t is plate thickness.

Ϭ= 6M/ t^2 (Equation 7)

As an example of using Roark’s formulas, consider the following scenario. A solid circular plate, 0.2 in thick and 20 inches in diameter, is simply supported along the edge and loaded with a uniformly distributed load of 3 PSI. It is required to determine the maximum stress, given E= 30$(10)^6$ PSI and ν= 0.285.

Applying Roark’s formulas results in a stress value of 9240 PSI at the plate center.

Applying Roark’s formulas to a scenario where the edges are fixed instead of simply supported result in a stress value of 3614 PSI (equations 5 and 7) at plate center and a stress value of 5625 PSI (equations 6 and 7) at the fixed edge of the plate.

Example of use of NASTRAN/PATRAN Finite Element software for stress analysis of a thin uniformly loaded circular plate:

The same geometry and loading conditions of the example problems solved using the classical stress analysis techniques are modeled using the NASTRAN/PATRAN Finite Element software, and the results follow. The details of developing the model and the theoretical details of the chosen elements (which are plate elements) are not decribed in this article.

Figure 2 shows the meshing, the boundary conditions, and the loading of the FE model.

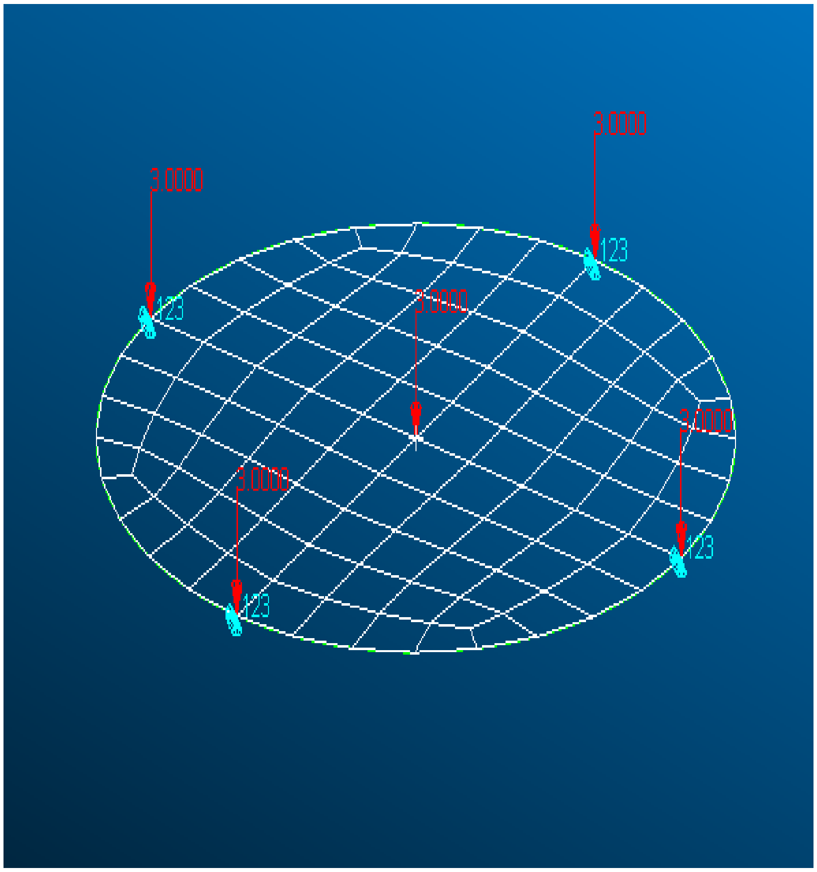

Figure 2: Finite Element Model of a 20 inch ID circular plate with a uniform thickness of 0.2 inches that is simply supported at its edge and is subjected to 3 PSI uniform pressure

Figure 3 shows the stress contour of the model of figure 2.

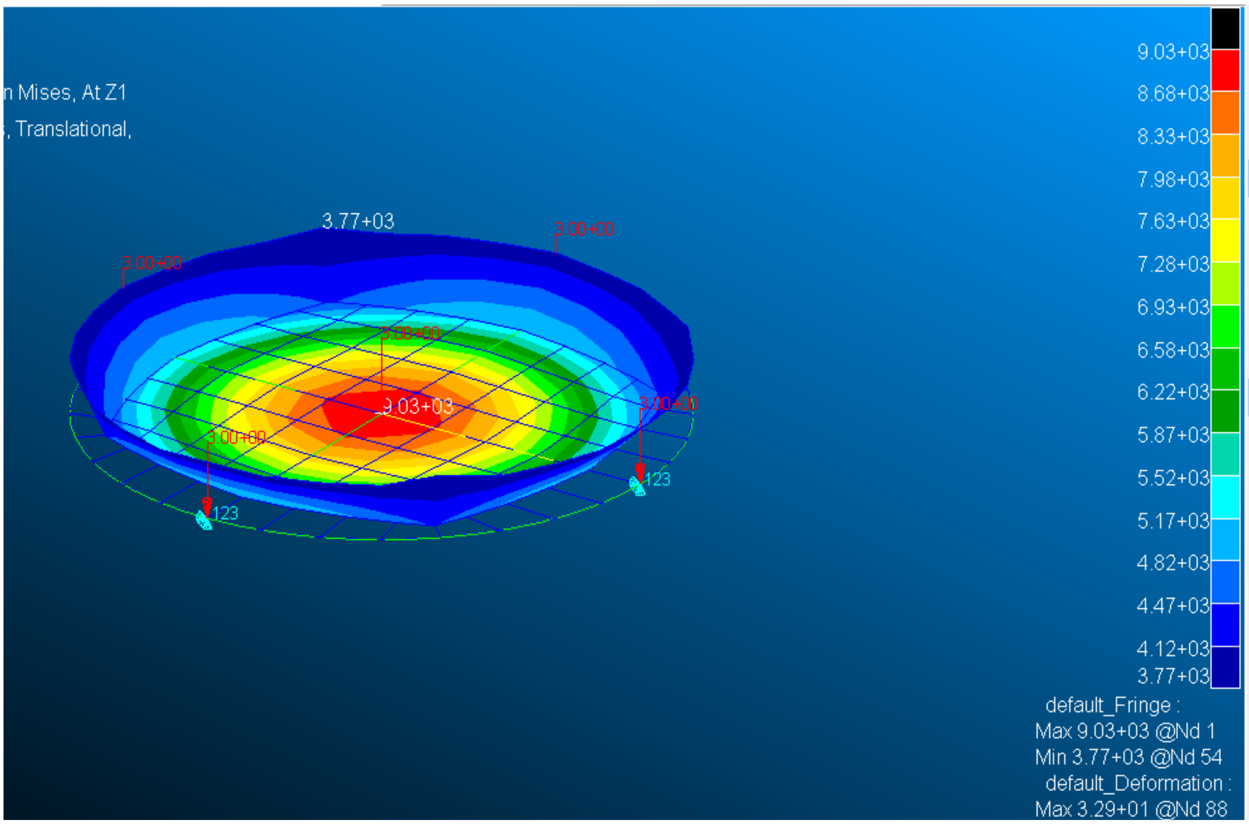

Figure 3: Stress contour of the model of figure 2

The maximum stress value obtained from the finite element model is at the plate center and is 9030 PSI. The stress value obtained from the Finite Element model is compatible with the result obtained from Roark's formulas (9240 PSI).

Figure 4 shows the model of figure 2, except that in the model of figure 4, the edges are fixed instead of simply supported.

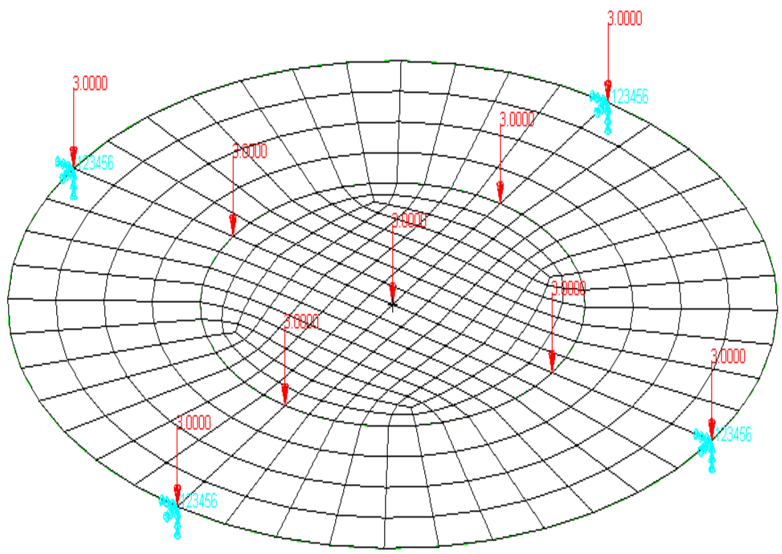

Figure 4: Finite Element Model of a 20 inch ID circular plate with a uniform thickness of 0.2 inch that is fixed at its edge and is subjected to 3 PSI uniform pressure

Figure 5 shows the stress contour obtained by the NASTRAN/PATRAN finite element model of figure 4.

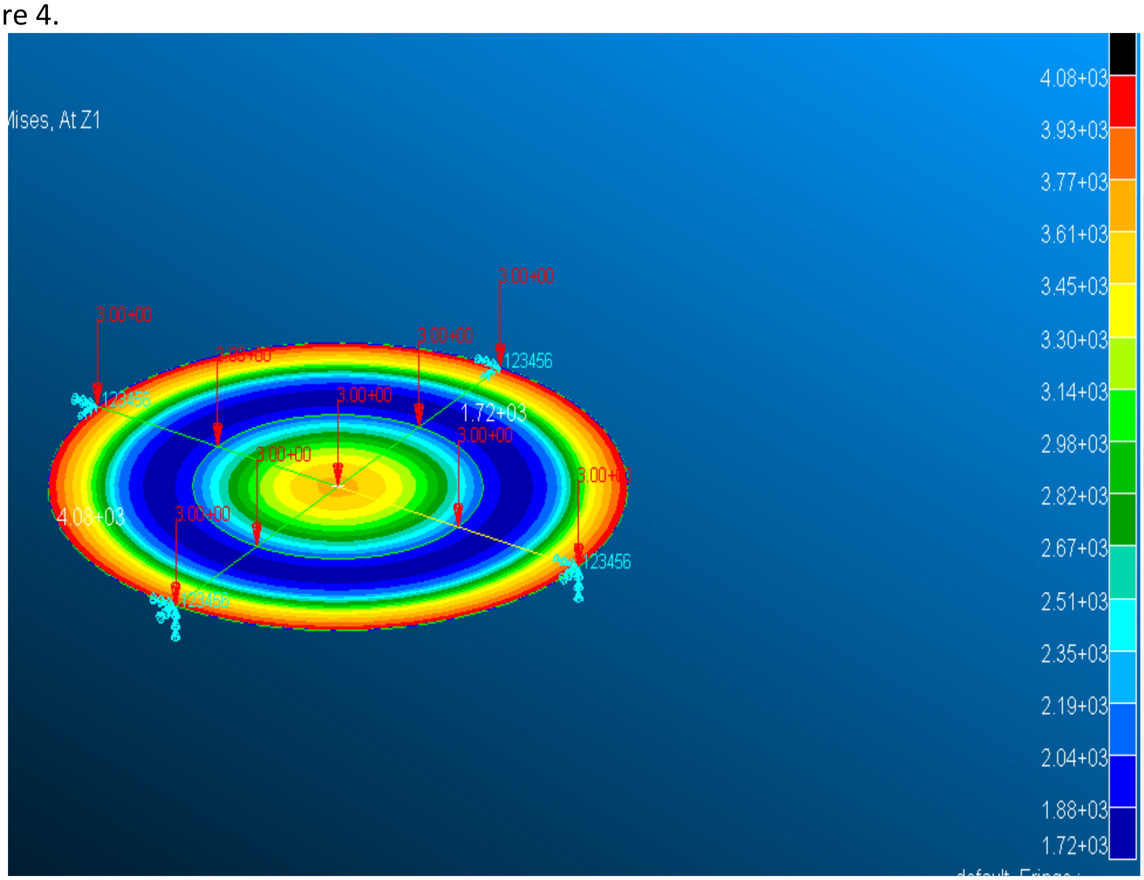

Figure 5: Finite Element stress contour obtained by NASTRAN/PATRAN for the Finite Element model of figure 4

The stress contour of figure 5 shows that the stress value at the plate center is 3610 PSI and the stress value at the fixed edge is 4080 PSI. This compares against a stress value of 3614 PSI at plate center and a stress value of 5625 PSI at the fixed edge of the plate obtained from Roark's formulas. It is accepted industry practice that if the correct FE modeling technique is being used, the results of the FE models are more accurate than Roark's results, even if they show a lower stress value.

Figure 6 meshes the same conditions as the model of figure 4, except that improper meshing techniques are used.

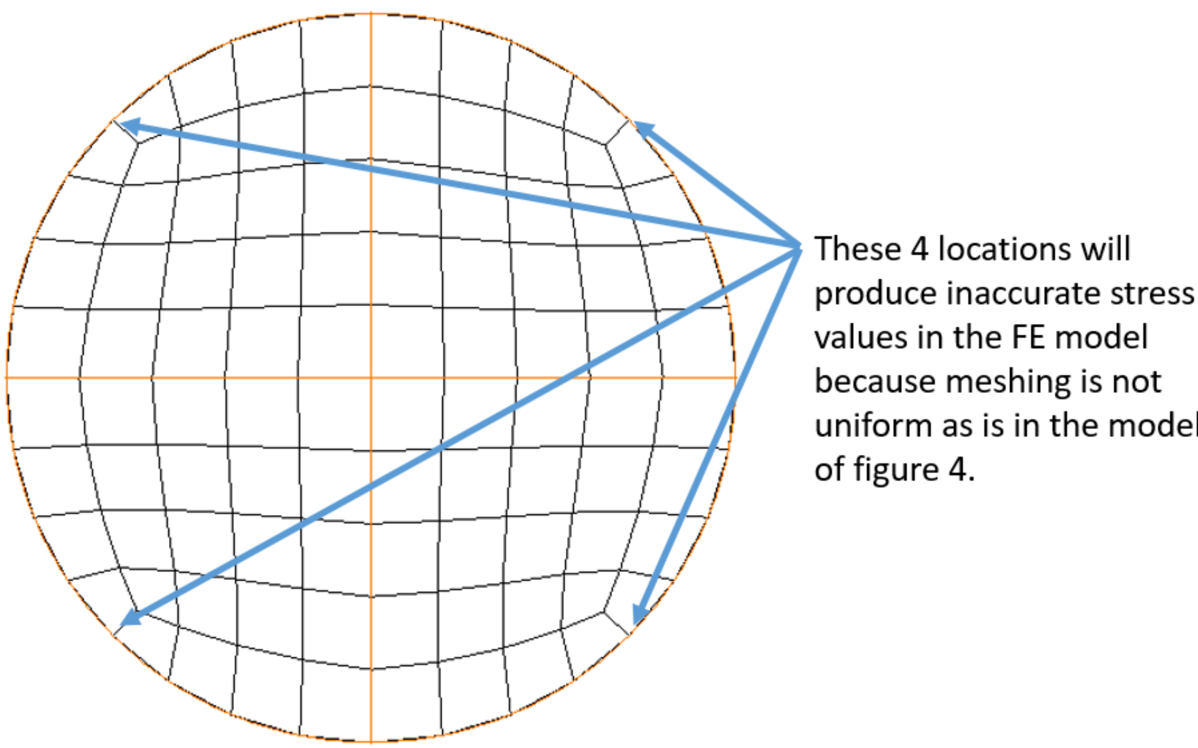

Figure 6: Demonstration of improper FE modeling practice that will result in incorrect stress values

Figures 7 and 8 show the FE results for the same geometry and loading conditions as the FE model of figure 5. Figure 8 also contains explanations as to how improper meshing techniques lead to incorrect results.

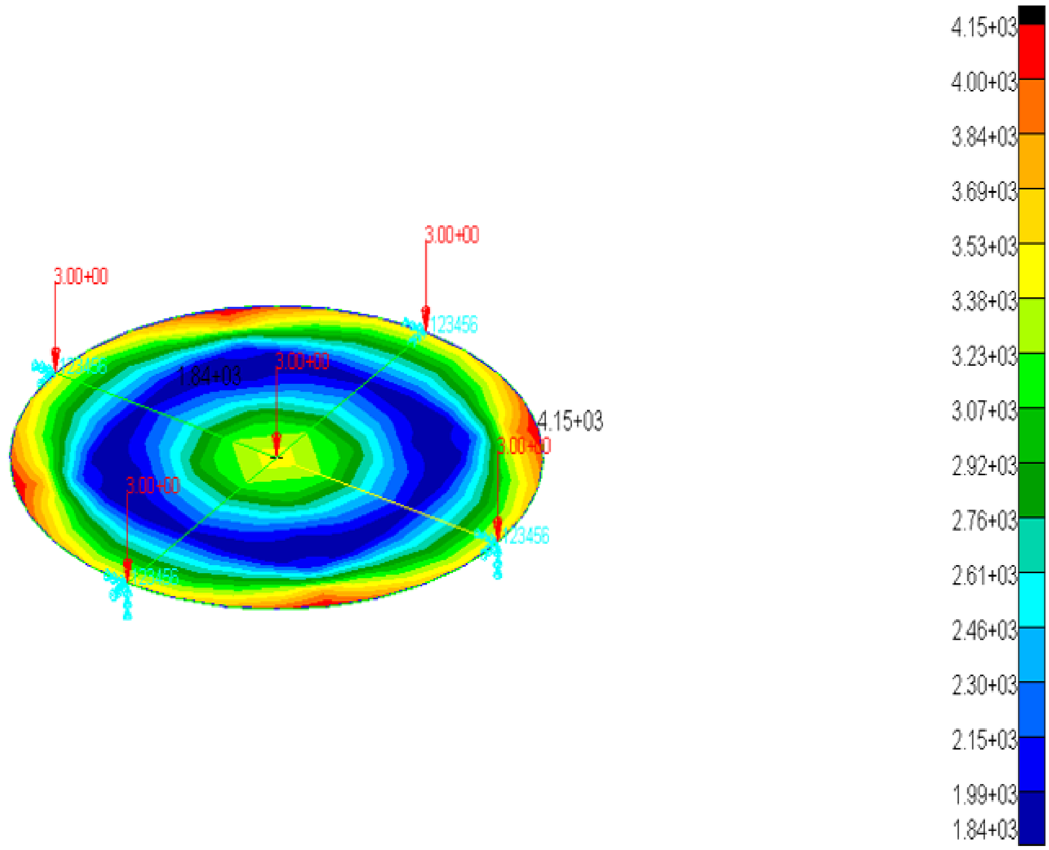

Figure 7: Stress contour for a FE model with conditions of the model of figure 4 but with improper meshing of figure 6

we have areas where inconsistent stress results are being calculated when we know stresses should be consistent. The red areas around the outer edge have higher stresses than the remaining edge areas. The reason stresses at the outer edge must be uniform is that the loading is a constant pressure throughout a circular plate and the outer edge has the same constraints all around the circle.

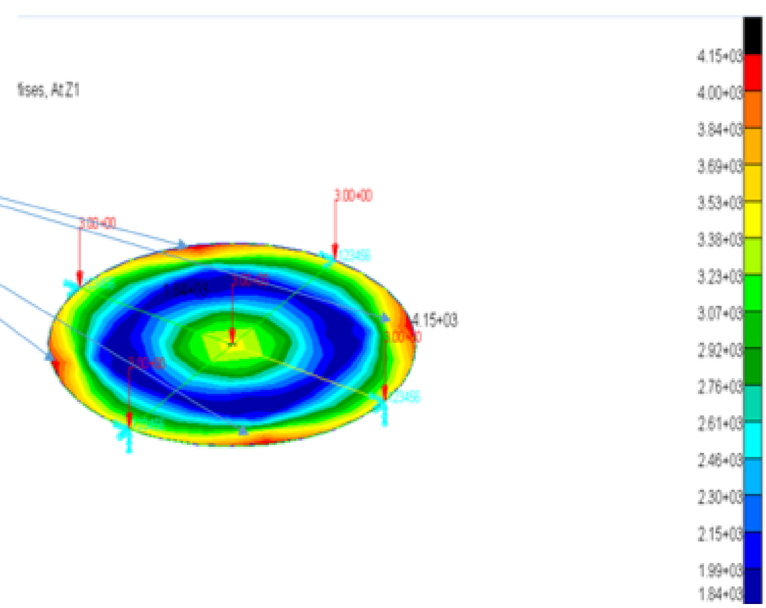

Figure 8: Same stress contour shown in figure 7 with

explanation for the presence of incorrect stress values

Figure 9 is a model of the geometry of figure 4, where a centric hole is added to the plate. All the remaining conditions remain the same as the model of figure 4. The modeling technique used in figure 9 is similar to that used in figure 4, meaning that the non-uniform meshing techniques used in figure 6 are avoided. Figure 10 shows the stress contour of the model of figure 9. Since the modeling technique of the geometry of figure 9 was verified by modeling techniques of figure 4, one can be assured that the results shown in figure 10 are accurate. There is a known classical solution for the condition of the model of figure 9. By comparing the stress values from the classical technique against the FE technique, it is concluded that, indeed the FE model is producing accurate results.

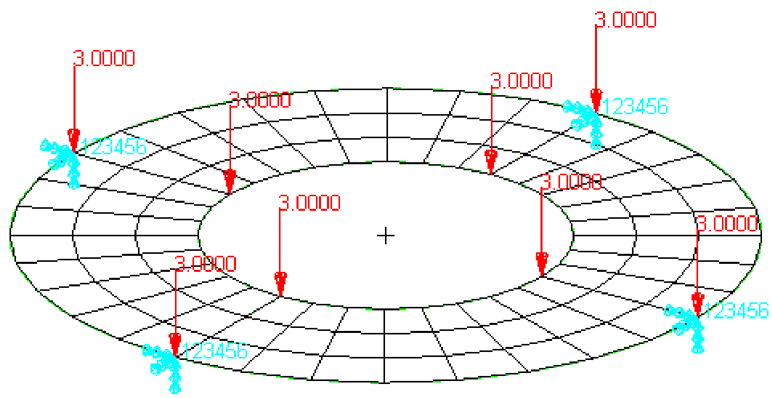

Figure 9: FE model of the model of figure 4 where a centric hole is added

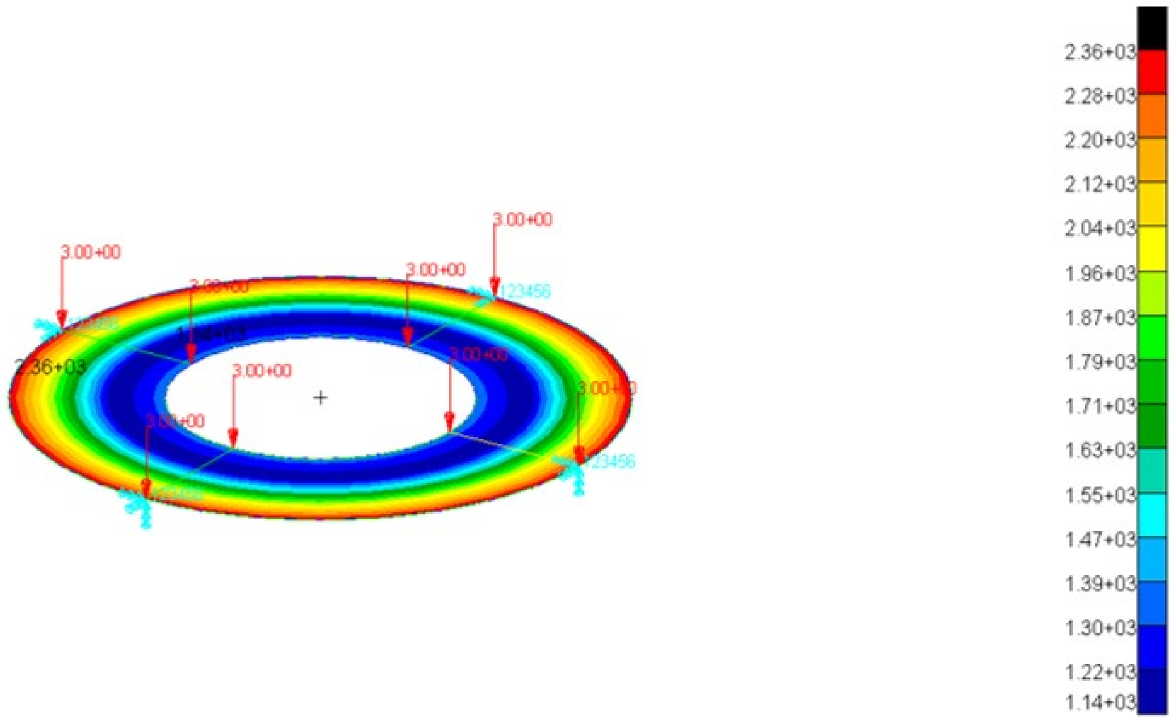

Figure 10: Stress contour of the FE model of figure 9

Conclusion & summary:

In the current article, it is shown that FE modeling can be a powerful tool that can be utilized to perform structural analysis for scenarios where no classical solution exists. It was also shown that improper FE modeling techniques produce incorrect results.

Using known solutions from Roark's formulas for stress and strain were demonstrated as a means of checking the accuracy of a FE model. The verified FE model was then used to analyze a condition where the model was changed. A change was chosen that has a known classical solution. The known classical solution verified that once a FE modeling technique is verified to be accurate for the conditions under study, minor changes can be made to the model while maintaining the modeling techniques, and the results will be accurate.

References:

[1] Roark's Formulas for Stress and Strain 6th edition, by Warren Young.
[2] Mechanics of Materials, 4th edition, By Higdon, Ohlsen, Stiles, Weese and Riley.
[3] A first Course in the Finite Element Method by Daryl L. Logan.
[4] User's guide for MSC.visualNastran for Windows.

Biographical Information:

Dr. Hagigat is teaching undergraduate and graduate engineering technology courses at The University of Toledo. Dr. Hagigat has an extensive industrial background, and he is continuously emphasizing the practical applications of engineering subjects covered in a typical engineering technology course.